Journal of Engineering Physics and Thermophysics, Vol. 80, No. 3, 2007

SIMULATION OF THE PROCESS OF MOISTURE TRANSFER IN THE ZONE OF AERATION ON THE BASIS OF THE CAPILLARY POTENTIAL EQUATION

N. D. Bobarykin

UDC 626.86:517.9(06)

A time-dependent model of the regime of moistening in the rootage layer of ameliorated soil is presented. It is based on the solution of a differential equation for the capillary potential of moisture transfer in the zone of soil aeration. The motion of moisture from the surface of groundwater into the layer of active moisture transfer of the aeration zone determines the aqueous-physical properties of soil, the moisture saturation degree of this layer, and the position of the groundwater level. The motion of soil moisture is characterized by an unsteady regime in which the moisture content changes not only with depth, but also in time.

A flux of moisture U feeding soil in the zone of aeration from the groundwater level is considered in [1], where the differential equation of steady-state motion of moisture in soil was used:

$$U = -\lambda \frac{\partial}{\partial z}(F - gz) . \tag{1}$$

The moisture conductivity coefficient can be represented by the exponential dependence [1]

$$\lambda = \lambda_0 \exp(wF) . \tag{2}$$

A constituent of the general time-dependent mathematical polder system model considered in [2] is the model [3] of the regime of moistening in the rootage layer of ameliorated soils, which is based on calculation of the capillary moisture-transfer potential in the aeration zone.

To calculate a moisture flux U, we write a partial differential equation for the capillary moisture potential F with allowance for Eq. (1) in the form [2, 4]

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial z}\left[\lambda \frac{\partial}{\partial z}(F - gz)\right] \tag{3}$$

or

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial F}{\partial z}\right) - gw\lambda \frac{\partial F}{\partial z} . \tag{4}$$

Initial and Boundary Conditions. As the initial conditions for the function of the moisture capillary potential F represented by Eq. (4) a linear function was specified so that on the soil surface ($z = H$) $F = 0$ and on the groundwater surface ($z = 0$) the value of the function F corresponded to a certain value of moisture flux equal to U_{max} (determined below).

The boundary conditions on the soil surface were prescribed depending on the values of total evaporation E, intensity of precipitation X, etc. In these calculations, evaporation fluxes E and those of atmospheric precipitation X

Kaliningrad State Technical University, 1 Sovetskii Ave., Kaliningrad, 236000, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 3, pp. 97–102, May–June, 2007. Original article submitted August 26, 2005; revision submitted October 6, 2006.

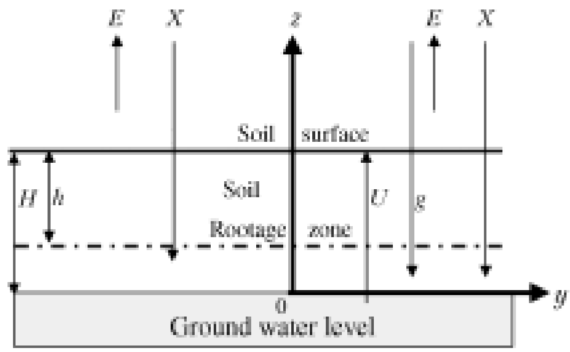


Fig. 1. Schematic diagram of soil-ground with indication of the coordinate system and acting fluxes of overall evaporation E, atmospheric precipitation X , and acceleration g.

were taken equal to zero. An explicit expression for the derivative of the capillary potential of moisture F over the coordinate z from Eq. (1) is written as follows:

$$\left.\frac{\partial F}{\partial z}\right|_{z=H} = \frac{E - X}{\lambda} - g \; . \tag{5}$$

From this relation it follows that the boundary conditions on the soil surface are prescribed in the form of the values of the derivative $\left.\frac{\partial F}{\partial z}\right|_{z=H}$ which takes into account the fluxes of total evaporation and atmospheric precipitation, as well as the gravitational potential.

The specification of the values of boundary conditions for the capillary potential at a ground water level is more complex than at the boundary with the soil [1, 2]. The author's investigations [1] of the value of a maximum moisture flux $U_{\max}$ with measurement of the potential F and other characteristics of heat and moisture conductivity and on the basis of the potential theory of moisture transfer have shown the following. The major factor which determines the properties of the soil of a drained area by maximum moisture transfer into the zone of aeration is the position of the level of ground water H and the depth of rootage zone h (see Fig. 1). Here, the highest possible moisture flux $U_{\max}$ into the aeration zone is approximately determined by the following dependence [1]:

$$U_{\max} = a\,(H - h)^3 + b\,(H - h)^2 + c\,(H - h) + d \; , \quad U_{\max} \geq 0 \; . \tag{6}$$

For peat soils of a hypnosegde back bog at peat decomposition of about 40% the values of these coefficients [1] are tabulated.

The capillary moisture potential on the surface of groundwater can be represented by the partial derivative $\left.\frac{\partial F}{\partial z}\right|_{z=0}$ as

$$\left.\frac{\partial F}{\partial z}\right|_{z=0} = \frac{U_{\max}}{\lambda_0} - g \; . \tag{7}$$

Numerical Methods of Solving the Equation. To solve the boundary-value problem (4), (5), (7), we will avail ourselves of the pivot method. From Eq. (4) we go over to finite-difference equations. For this purpose, we discretize the problem, i.e., we introduce uniform grids in the variables z and t:

$$0 = z_0 < z_1 < \ldots < z_{n-1} < z_n = H \; , \tag{8}$$

where $z_i = i\delta$, $\delta = H/n$, $i = 0, 1, ..., n$. We define $F_i = F(z_i)$:

$$0 = t_0 < t_1 < ... < t_{k-1} < t_k = T, \tag{9}$$

where $t_j = j\tau$, $\tau = T/k$, $j = 0, 1, ..., k$. We define the value of the moisture potential at the ith node and on the jth time layer as $F_{ij} = F(z_i, t_j)$.

We replace the initial partial differential equation of parabolic type (4) by a finite-difference one at inner nodes according to the divergent scheme (on the difference scheme the conservation laws hold):

$$\frac{F_{i,j+1} - F_{i,j}}{\tau} = \frac{1}{2\delta^2}\Big[(\lambda_{i+1} + \lambda_i)(F_{i+1,j+1} - F_{i,j+1}) - (\lambda_i + \lambda_{i-1})(F_{i,j+1} - F_{i-1,j+1})\Big] - g\lambda_i w \frac{F_{i,j+1} - F_{i-1,j+1}}{\delta},$$

where $i = 1, 2, ..., n-1$, $j = 0, 1, ..., k-1$. These equations are brought to the explicit canonical three-diagonal form

$$A_i F_{i+1,j+1} - B_i F_{i,j+1} + C_i F_{i-1,j+1} = -D_i, \tag{10}$$

where $A_i = \dfrac{(\lambda_i + \lambda_{i+1})}{2\delta^2}$; $C_i = \dfrac{(\lambda_i + \lambda_{i-1})}{2\delta^2} - \dfrac{g\lambda_i w}{\delta}$; $B_i = A_i + C_i + \dfrac{1}{\tau}$; $D_i = \dfrac{F_{i,j}}{\tau}$.

We note that the stability criterion of computation by the pivot method against errors of rounding is met, since

$$B_i - (A_i + C_i) = \frac{1}{\tau} > 0, \tag{11}$$

and this means that the resulting divergent difference scheme (10), which describes the initial differential equation (4), is by all means stable and approximates it with the first order of accuracy in time and with the second one over the coordinate.

The solution of the equation for the capillary potential (4) after it was reduced to a tridiagonal form (10) is realized [2, 3] by the pivot method. Here, the sought-for discrete function $F_{i,j+1}$ is calculated from the recurrent formula (oppositely directed pivots)

$$F_{i,j+1} = Q_{i+1} F_{i+1,j+1} + P_{i+1}, \quad i = n-1, ..., 1, 0, \tag{12}$$

where the pivot coefficients Q_{i+1} and P_{i+1} are determined with the direct course $i = 1, ..., n-1$:

$$Q_{i+1} = \frac{A_i}{B_i - C_i Q_i}, \quad P_{i+1} = \frac{C_i P_i + D_i}{B_i - C_i Q_i}. \tag{13}$$

As follows from recurrent relations (13), for the start of calculation we must have the values of Q_1 and P_1, which are determined with the aid of the left boundary condition (7) at z_0:

$$\frac{F_{1,j+1} - F_{0,j+1}}{\delta} = \frac{U_{\max}}{\lambda_0} - g \quad \text{or} \quad F_{0,j+1} = F_{1,j+1} - \delta\left(\frac{U_{\max}}{\lambda_0} - g\right). \tag{14}$$

The coincidence of the recurrent relation (12) at $i = 0$ with expression (14) is satisfied only under the following conditions:

$$Q_1 = 1 \quad \text{and} \quad P_1 = -\delta\left(\frac{U_{\max}}{\lambda_0} - g\right). \tag{15}$$

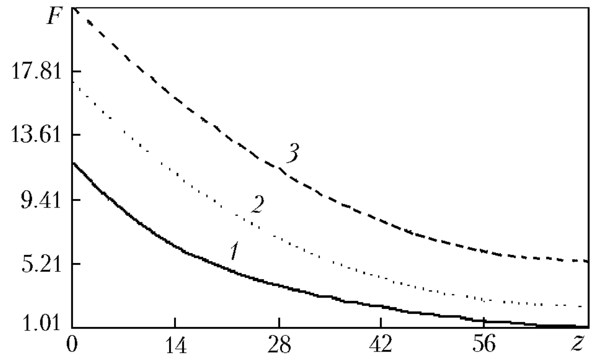


Fig. 2. Calculated dependences of capillary potential F on the height z for three time moments: 1) 1.5 day; 2) 5 days; 3) 10 days. F, cm^2/day^2; z, cm.

The right boundary condition (5) at z_n is used for the reverse course of $i = n-1, ..., 1, 0$ from Eq. (12), as a result of which the sought-for function of the capillary potential of moisture F is calculated:

$$\frac{F_{n,j+1} - F_{n-1,j+1}}{\delta} = \frac{E - X}{\lambda_n} - g \, . \tag{16}$$

By simultaneously solving the system of equations formed by relations (16) and (12) at $i = n - 1$, we determine the value of the capillary potential $F_{n,j+1}$ on the right boundary of the computational domain and the next time layer:

$$F_{n,j+1} = \frac{\delta\left(\frac{E - X}{\lambda_n} - g\right) + P_n}{1 - Q_n} \, . \tag{17}$$

The given algorithm can be easily realized in any algorithmic language and is the computationally stable, most effective, and economic method of solution of partial differential equations of parabolic type and takes into account the initial and boundary conditions. Using Eq. (1), from the calculated space-time distribution of the capillary potential F we may calculate the values of the flux of moisture:

$$U_i = -\lambda_i \frac{F_{i+1,j+1} - F_{i-1,j+1}}{2\delta} + \lambda_i g \, , \quad i = 1, ..., n-1 \, . \tag{18}$$

The values of the moisture flux U at boundary points at $z = 0$ and $z = H$ are also calculated from Eq. (18) on replacing the central difference derivative by the left and right difference products, respectively.

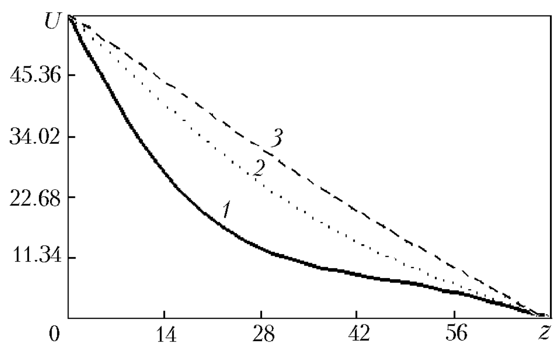
Results of Numerical Calculations. In carrying out numerical calculations of the values of the capillary potential F and moisture flux U, the most complex is the assigning of the numerical values of the moisture conductivity coefficient λ [1, 4], which depend on the properties of the soil and the amount of moisture stored in it. The conductivity potential coefficient (moisture diffusion in the zone of aeration) determines the property of the soil-ground of the aeration zone and serves as the coefficient of proportionality between the moisture flux density and humidity gradient. In [5], for peat soils the value of the coefficient of filtration K_f is approximately equal to 5.0–0.2 m/day, whereas the value of the moisture conductivity coefficient λ lies in the range 0.020–0.003 m^2/day.

In the present work the moisture conductivity coefficient λ was assigned, just as in [1], in the form of the exponential dependence (2), with $\lambda_0 =$ 101.0 cm^2/day. The calculations were performed for the conditions of shallow bogs underlined by fine-grained sand (all cultures) [1] with the ground water level H = 70 cm and depth of rootage zone h = 40 cm.

Figure 2 presents the calculated values of the capillary potential of moisture F for three moments of time: 1) 1.5 day; 2) 5 days; 3) 10 days. Equation (18) allows one to calculate the values of moisture flux U into the aeration

TABLE 1. The Values of the Coefficients in Eq. (6)

Character of the surface	*a*	*b*	*c*	*d*	Limitations	
					$\xi = H - h$	U_0
Open soil	–0.0007	0.102	–5.37	111.2	$10 \geq \xi \leq 65$	67
Deep bogs:						
springcrops	0	0.0113	–2.26	115.0	$10 \geq \xi \leq 95$	94
perennial herbs	0.00015	–0.02	–0.6	90.0	$0 \geq \xi \leq 90$	90
potatoes	–0.0005	0.092	–5.78	134	$10 \geq \xi \leq 90$	85
Shallow bogs underlay a by a finely granular sand	0.0001	–0.02	–0.6	90.0	$0 \geq \xi \leq 90$	90



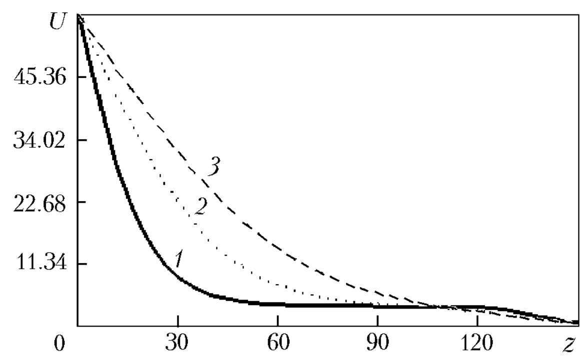


Fig. 3. Calculated dependences of moisture fluxes U corresponding to the capillary potential values given in Fig. 2. The values for 1–3 are the same as in Fig. 2. U, cm^3/day^3; z, cm.

Fig. 4. Calculated dependences of moisture fluxes U on the coordinate z for the groundwater level H = 150 cm. The values for 1–3 are the same as in Fig. 2. U, cm^3/day^3; z, cm.

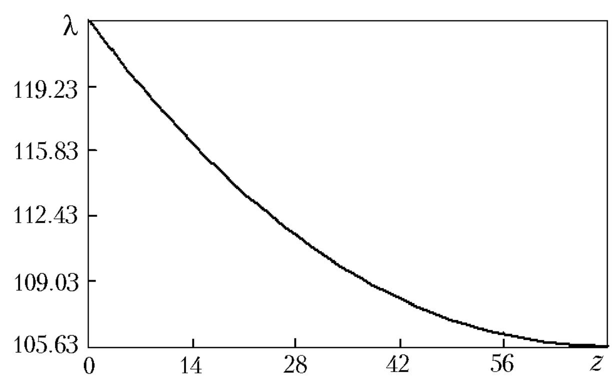
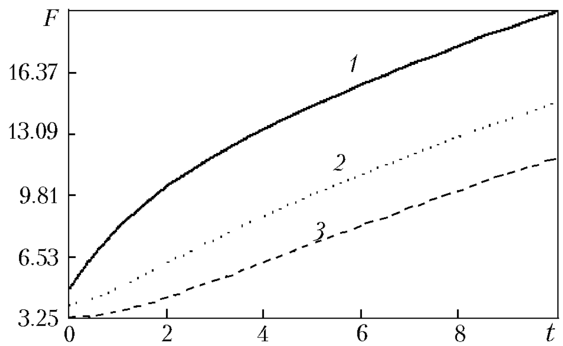


Fig. 5. Calculated values of the moisture conductivity coefficient. λ, mm^2/day; z, cm.

zone. The distributions of the moisture flux U over the height for the three time moments are presented in Fig. 3. The value of the moisture flux feeding the rootage zone U_0 = 56.7 cm^3/day^3 at the groundwater level (at z = 0) was calculated from Eq. (6), with the values of the coefficient for this equation being taken from Table 1.

It should be noted that at a constant groundwater level H and constant feeding moisture flux U_0 from the groundwater level, the values of the capillary potential F and of the moisture flux U increase with time (see Figs. 2 and 3). The capillary potential function F of the variable z has an explicit exponential character (Fig. 2). At the same



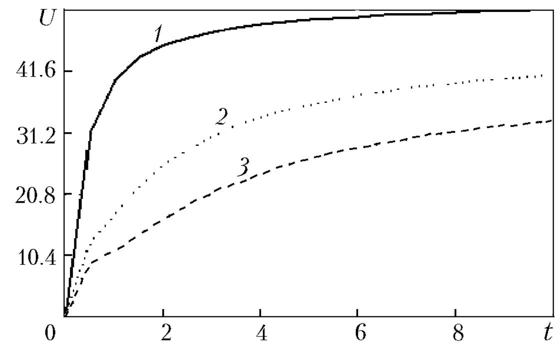


Fig. 6. Time dependence of the capillary potential F for three values of the height z: 1) 1.75; 2) 17.5; 3) 26.25 cm. F, cm^2/day^2; z, cm; t, day.

Fig. 7. Time dependence of the moisture fluxes U for three values of the height z: 1) 1.75 cm; 2) 17.5; 3) 26.25 cm. U, cm^3/day^3; z, cm; t, day.

time, the feeding flux of moisture U during the first day also has an exponential dependence, but already by the beginning of the 10th day it becomes linear (Fig. 3, curve 3), which is attributable to the isotropicity of the soil-ground.

To elucidate the role of the groundwater level in the distribution of moisture flux over the height we will consider the results of calculations of moisture flux U for the groundwater level H = 150 cm which are presented in Fig. 4 (in Fig. 3 H = 70 cm). We note that in these calculations the feeding flux of moisture U_0 from the groundwater level was prescribed the same as for calculations with the groundwater level equal to H = 70 cm (see Fig. 3).

As was expected, a comparison of the height dependences of moisture fluxes U for different values of groundwater levels which are presented in Figs. 3 and 4 (in the rootage zone 0–40 cm) shows that the redistribution of the moisture flux over the height occurs with an appreciable decrease in the values of U in the groundwater level, and thus there occurs depletion of moisture in the soil of the rootage zone. The capillary transfer of moisture from the groundwater level into the aeration zone is determined not only by the position of groundwater, but also by the value of the moisture conductivity coefficient λ. However, it should be noted that the structure of the dependence of moisture flux on different positions of the groundwater level for different numerical values of the moisture conductivity coefficient is preserved, and only numerical values of the flux of moisture transfer are changed.

As it follows from the analysis of the height dependence of λ given in Fig. 5, here the height behavior of the capillary potential F is repeated (see Fig. 2) with account for its exponential dependence. At the present time there is very limited information on the numeral values of the moisture conductivity coefficient [1–3] for various soil-grounds, and further experimental investigations in this area are needed. Figures 6 and 7 present the time dependences of the capillary potential F and moisture fluxes U for different heights from the groundwater level. It is seen from these figures that the values of F and U increase with time exponentially for all the values of z.

The feeding moisture flux U from the surface of the groundwater level into the aeration zone in the first day (see Fig. 7) increases to the utmost and then attains some stationary values which are different for different heights.

Thus, water catchment areas of polder systems are distinguished by small areas and great bogging of the surface and ground sinks from cultivated bogs. Investigations on water catchment areas of the drainage system of Belarus show that in frosty winters low cultivated bogs freeze through to a great depth [1]. During mild winters and with deep snow cover the depth of freezing decreases considerably, and often there is no continuous freezing, and by the beginning of snow melting the lowest groundwater levels are set as well as a large accumulating capacity in the aeration zone, the presence of which may retain up to 100–170 mm of melt water [6]. In years with slight freezing of peat soil with sufficient water penetrability and moisture capacity the melt water is spent to feed soil and groundwaters, and the spring sink is mainly formed as a ground sink and does not lead to heavy floods. Under the conditions of frozen soil and intense snow melting an intense surface sink is observed, and the draining network of polder systems favors more rapid sink into conducting open channels, thus increasing the maximum flow rate in them. Therefore, it is important to be able to solve the problems of mathematical simulation and prediction of the saturation of soil with moisture not only from the viewpoint of the vegetation of plants but also formation of sinks from the areas being drained [7].

This work was carried out with support from the Russian Foundation for Basic Research, project No. 06-01-00396-a.

NOTATION

A_i, B_i, C_i, D_i, coefficients; E, moisture flux spent on evaporation, cm^3/day^3; F, capillary potential of soil moisture, cm^2/day^2; g, free fall acceleration, cm/day^2; H, groundwater level, cm; h, depth of rootage zone, cm; i, number of spatial node; j, number of a temporal layer; K_f, coefficient of filtration, cm/day; k, number of temporal layers; n, number of spatial nodes; P, Q, pivot coefficients of the equation given; T, final time of the process; t, current time of the process; U, moisture flux, cm^3/day^3; U_0, constant feeding flux of moisture from groundwater, cm^3/day^3; U_{max}, maximum moisture flux, cm^3/day^3; w, constant for the given type of soil, day^2/cm^2; X, flux of moisture from atmospheric precipitation, cm^3/day^3; z, coordinate of the axis by which moisture flux (from groundwater level to soil surface) is calculated, cm; δ, step of spatial computational grid; λ, moisture conductivity coefficient, cm^2/day; λ_0, moisture conductivity coefficient at $F = 0$, cm^2/day; τ, step of temporal computational grid. Subscripts: max, maximum; f, filtration.

REFERENCES

1. V. F. Shebeko, P. I. Zakrzhevskii, and É. A. Bragilevskaya, *Hydrologic Calculations in Designing Draining and Draining-Moistening Systems* [in Russian], Gidrometeoizdat, Leningrad (1980).
2. N. D. Bobarykin, *Optimal Control of the Regime of Groundwater Based on the Invariant Nonstationary Mathematical Model of Polder Systems* (Scientific Publication) [in Russian], KGTU, Kaliningrad (2004).
3. N. D. Bobarykin, Calculation of moisture exchange with groundwater on the basis of the differential equation for capillary potential, in: *Coll. Sci. Papers of the Kaliningrad State Technical University "Mathematical Simulation and Numerical Methods Solving Integral-Differential Equations"* [in Russian], Kaliningrad (2003), pp. 19–30.
4. V. F. Shebeko, *Evaporation from Bogs and Soil Moisture Balance* [in Russian], Urozhai, Minsk (1965).
5. A. V. Luikov, *Heat and Mass Transfer: Handbook* [in Russian], 2nd rev. and augm. edn., Énergiya, Moscow (1978).
6. V. F. Shebeko, *Change in the Microclimate under the Influence of Melioration of Bogs* [in Russian], Nauka i Tekhnika, Minsk (1977).
7. V. F. Shebeko, *Hydrologic Regime of Drained Territories* [in Russian], Urozhai, Minsk (1970).